MATH 212 - QUIZ 1

October 27, 2010

Exercise 1. *Consider the 2π-periodic function $f(x) = |\sin x|$ on $]-\pi, \pi[$.*

   a) *To which class of functions does f belong?*

   b) *Showing the details of your work, find the Fourier coefficients a_0, a_n and b_n and then the Fourier series of the function f, knowing that*

$$\sin\theta\cos\phi = \frac{1}{2}\Big(\sin(\theta+\phi) + \sin(\theta-\phi)\Big)$$

$$and \ \sin\theta\sin\phi = \frac{1}{2}\Big(\cos(\theta-\phi) - \cos(\theta+\phi)\Big).$$

Exercise 2. *Consider the function f given on $[0, \pi]$ by $f(x) = x(\pi-x)$.*

   a) *Find the odd extension g_1 and the even extension g_2 of f to $[-\pi, \pi]$.*

   b) *By half-range expansions, find the Fourier sine series and the Fourier cosine series of the function f.*

Exercise 3. *If $f(x)$ and $g(x)$ have period p.*

   a) *Find the period of $h(x) = af(x) + bg(x)$ (a and b are any constants).*

   b) *Find the periods of $f(ax)$ and of $f(\frac{x}{b})$ (a and b are any constants $\neq 0$).*