If people do not believe that mathematics is simple, it is only because they do not realize how complicated life is. ~John Louis vo Neumann

## MATH 212 - QUIZ 1

## October 27, 2010

Exercise 1. Consider the $2 \pi$-periodic function $f(x)=|\sin x|$ on $]-\pi, \pi$ [.
a) To which class of functions does $f$ belong?
b) Showing the details of your work, find the Fourier coefficients $a_{0}, a_{n}$ and $b_{n}$ and then the Fourier series of the function $f$, knowing that

$$
\begin{aligned}
\sin \theta \cos \phi & =\frac{1}{2}(\sin (\theta+\phi)+\sin (\theta-\phi)) \\
\text { and } \sin \theta \sin \phi & =\frac{1}{2}(\cos (\theta-\phi)-\cos (\theta+\phi))
\end{aligned}
$$

Exercise 2. Consider the function $f$ given on $[0, \pi]$ by $f(x)=x(\pi-x)$.
a) Find the odd extension $g_{1}$ and the even extension $g_{2}$ of $f$ to $[-\pi, \pi]$.
b) By half-range expansions, find the Fourier sine series and the Fourier cosine series of the function $f$.

Exercise 3. If $f(\bar{x})$ and $g(x)$ have period $p$.
a) Find the period of $h(x)=a f(x)+b g(x)$ ( $a$ and $b$ are any constants).
b) Find the periods of $f(a x)$ and of $f\left(\frac{x}{b}\right)$ ( $a$ and $b$ are any constants $\neq 0$ ).

